The Investment Decision – Net Present Value and Internal Rate of Return

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1.0 Introduction

Within a business, the role of the Financial Manager is to make decisions that enable the business' objectives to be achieved in as financially efficient a manner as possible. One subset of these decisions is the investment of funds into different assets. These decisions, often referred to collectively as capital budgeting or "the investment decision", represent the most important decisions that the Financial Manager must make, as they determine the sources and extent of a business' cash flows, as well as its ability to service its debt and provide a return to its shareholders. As such, it is vital that Financial Managers use the appropriate methods to make these decisions, but also remain cognisant of the relative strengths and weaknesses of these methods. The aim of this article is to explain and critically compare the following two methods commonly used to financially appraise the investment opportunities available to a firm[1]:

  i.   Net Present Value
  ii.   Internal Rate of Return

2.0 Net Present Value

The Net Present Value (NPV) method of investment appraisal assesses the financial viability of a project by comparing the project's cash inflows to its cash outflows in a manner that accounts for the time value of money. Specifically, the NPV of a project is the sum of the present values of a project's periodic[2] cash flows, where periodic cash flows are discounted back to present values using an appropriate discount rate[3]. Mathematically, this can be expressed as follows:

$$NPV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots + \frac{C_t}{(1+r)^t}$$

Where:
C_0 to C_t are the project cash flows.
t=0 to t=t are the periods in which cash flows exist e.g. year 0 to year t.
r is the rate at which project cash flows are discounted

If the NPV of the project is positive, the project is deemed financially viable, as a positive NPV arises when the average periodic compound rate of return earned on the project's capital outlay is greater than the discount rate applied i.e. the rate of return on the project is greater than the required rate of return on the project. If the NPV of the project is negative, the project is deemed to be financially

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[1] Hereafter, investment opportunities available to a firm are referred to as "projects".
[2] Whilst the periodicity of a project's cash flows can vary i.e. quarterly, semi-annual or annual, the general approach is to present a project's cash flows on an annual basis.
[3] The discount rate used will depend on the riskiness of the project, the manner in which the project is to be financed, and the impact the project and its financing will have on the firm's existing operations and financing. It represents the project's required average periodic compound rate of return and is usually expressed as an annual percentage rate. It is often referred to as the "cost of capital".

unviable, as a negative NPV arises when the average periodic compound rate of return earned on the project's capital outlay[4] is less than the discount rate applied i.e. the rate of return on the project is less than the required rate of return on the project. The following example illustrates how the NPV method is used in practice and what it implies about the return generated by a project.

Example 1: NPV
ABC Ltd has the opportunity to invest in a project with the following forecasted annual cash flows:

Year	0	1	2	3	4
Cash Flow	-€2,000,000	€600,000	€800,000	€700,000	€650,000

The Financial Manager of ABC Ltd has determined that the appropriate cost of capital (COC) to apply as the discount rate to the project's cash flows is 12% p.a. The NPV of the project is:

$$-€2,000,000 + \frac{€600,000}{1.12^1} + \frac{€800,000}{1.12^2} + \frac{€700,000}{1.12^3} + \frac{€650,000}{1.12^4} = €84,802$$

As the NPV of the project is positive, the project is generating a rate of return greater than 12% i.e. given the cash inflows the project generates, on an equivalent basis the project is generating an annual rate of return in excess of 12% on the €2,000,000 invested. Hence, the project is deemed to be financially viable. If the NPV had been negative, it would have implied that the project is generating a rate of return less than 12% p.a. and it would be deemed financially unviable. If the NPV had been exactly zero, it would have implied that the project is generating a rate of return exactly equal to 12% p.a.

3.0 Internal Rate of Return
The NPV method informs the Financial Manager whether the rate of return on a project is greater or lesser than the COC used as the discount rate, but it does not tell the Financial Manager the actual value of the rate of return on a project. As Financial Managers must often report on a project's financial viability to individuals who may not fully grasp the concept of NPV and what it implies, Financial Managers may wish to identify the rate of return a project generates, as this is more easily communicated to and understood by individuals without the necessary background in finance. As noted above, a zero NPV implies the rate of return on a project is equal to the discount rate applied. Thus, one way to measure the rate of return on a project is to find the discount rate, that if applied to a project's cash flows, would result in the project having a zero NPV. The discount rate that results in a zero NPV is referred to as the internal rate of return (IRR). Once identified, this rate can be compared to the required rate of return on the project, and if it exceeds this rate i.e. IRR > COC, the project is deemed financially viable. If IRR < COC, the project is deemed financially unviable.

3.1   IRR and Linear Interpolation
As mentioned above, the IRR is the discount rate that results in a project having a zero NPV. Mathematically, this can be expressed as follows:

$$0 = C_0 + \frac{C_1}{(1 + IRR)^1} + \frac{C_2}{(1 + IRR)^2} + \frac{C_3}{(1 + IRR)^3} \dots + \frac{C_t}{(1 + IRR)^t}$$

Where:
C_0 to C_t are the project cash flows
t=0 to t=t are the periods in which cash flows exist e.g. year 0 to year t.

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[4] Hereafter, average periodic compound rate of return earned on the project's capital outlay is referred to as "rate of return on the project".

The nature of this expression means that it is impossible to algebraically manipulate it to isolate IRR on one side of the expression i.e. it cannot be rearranged to solve for IRR. Instead, a method known as linear interpolation can be used to approximate IRR, where multiple iterations of the method will result in increasingly more accurate approximations. To understand how linear interpolation works, we must first understand the relationship between a project's NPV and the discount rate applied to the project's cash flows.

Consider a project with conventional cash flows, where the project involves an initial cash outflow followed by a stream of cash inflows i.e. the signs of the project's cash flows overtime would show the following pattern: - + + + … +. The relationship between this project's NPV and the discount rate applied to its cash flows would appear as in Figure 1 if the NPV was plotted on the Y-axis and the corresponding discount rate (dr) was plotted on the X-axis[5]:

Figure 1
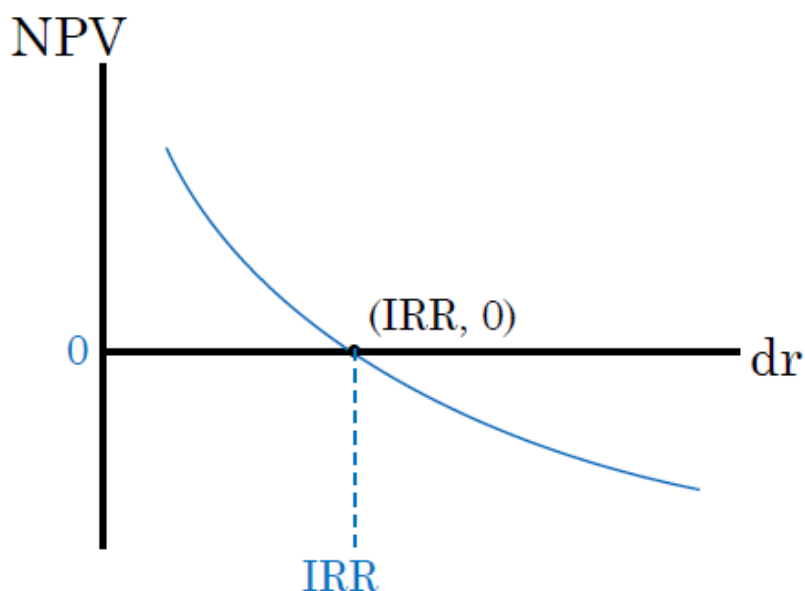
The relationship between NPV and dr is clearly curvilinear, and this is what makes solving for the IRR difficult. If the relationship was linear, we could use the rules of linear algebra to solve for the IRR relatively easily. However, for a project with conventional cash flows, the relationship is close to being linear, and if we assume that it is linear, we can use the rules of linear algebra to approximate the IRR. To see how, consider again the same conventional cash flow project in Figure 2, but now identify two other points on the curve, one with a discount rate that results in a positive NPV value, call it (r_1, NPV_1), and one with a discount rate that results in a negative NPV value, call it (r_2, NPV_2). If we draw a straight line through these two points, we see that it crosses the X-axis at a point adjacent to the IRR, let's call this point (IRR_{e1}, 0), where e1 stands for estimate one.

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[5] That is, if the NPV of the project was calculated numerous times using different discount rates, and the resulting pairs of discount rates and NPVs were plotted as points on an XY axis and the points joined up to form a curve.

Figure 2
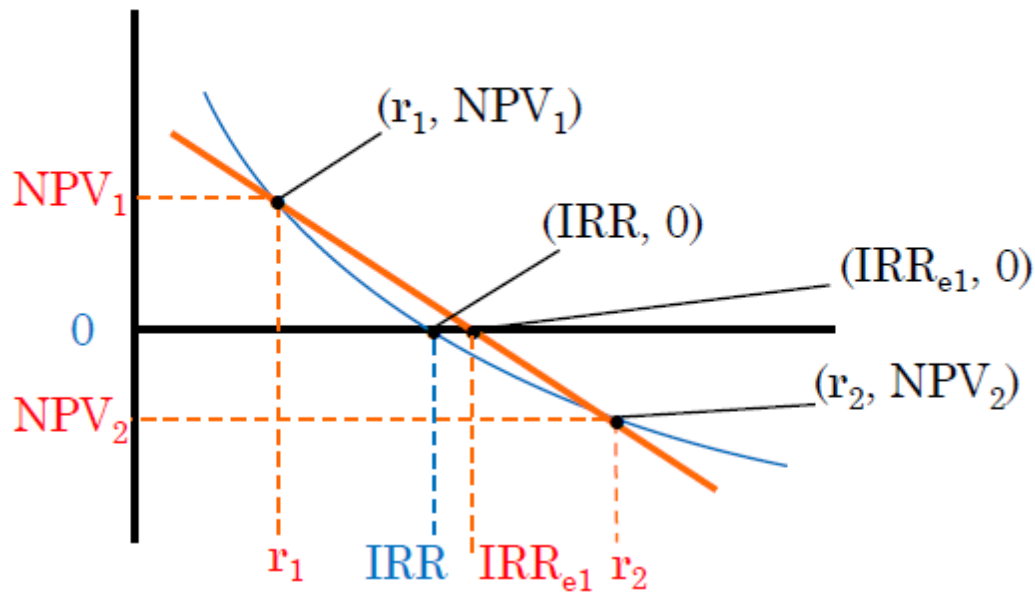
Given that this point lies on the same line as (r_1, NPV_1) and (r_2, NPV_2), we can find the value of IRR_{e1} and use this as an approximation of IRR. This can be achieved by recognising that the slope of the line between points (r_1, NPV_1) and (r_2, NPV_2) is the same as the slope of the line between points (r_1, NPV_1) and (IRR_{e1}, 0). Applying the formula for the slope of a line i.e.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

to our two sets of points, and setting them equal to each other, we get:

$$\frac{NPV_1 - NPV_2}{r_1 - r_2} = \frac{NPV_1 - 0}{r_1 - IRR_{e1}}$$

Rearranging this equation to isolate IRR_{e1} we get the following equation:

$$IRR_{e1} = r_1 + \frac{(r_2 - r_1) \times NPV_1}{NPV_1 - NPV_2}$$

This is the linear interpolation equation commonly used to approximate the IRR of a project[6]. The following example illustrates how linear interpolation is used in practice to approximate a project's IRR.

Example 2: IRR

In example 1 above, we see that ABC's project has a positive NPV of €84,802 and is deemed financially viable. The Financial Manager could also have reached this conclusion by using linear interpolation to approximate the project's IRR and then compare it to the COC. First, through trial and error, the Financial Manager would identify a discount rate that results in a positive NPV and a discount rate

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[6] The equation shown here is based on the notation used in this article. Other sources e.g. finance textbooks, may show a variation of this equation with different notation, but it is essentially the same equation.

that results in a negative NPV. For example, a discount rate of 7% results in an NPV of €326,789, and a discount rate of 17% results in an NPV of -€118,300.

$$-€2,000,000 + \frac{€600,000}{1.07^1} + \frac{€800,000}{1.07^2} + \frac{€700,000}{1.07^3} + \frac{€650,000}{1.07^4} = €326,789$$

$$-€2,000,000 + \frac{€600,000}{1.17^1} + \frac{€800,000}{1.17^2} + \frac{€700,000}{1.17^3} + \frac{€650,000}{1.17^4} = -€118,837$$

Letting (7%, €326,789) be (r_1, NPV_1) and (17%, -€118,837) be (r_2, NPV_2), and plugging these figures into the linear interpolation formula, we get an approximation for the IRR of the project of:

$$IRR_{e1} = 7\% + \frac{(17\% - 7\%) \times €326,789}{€326,789 - -€118,837} = 14.33\%$$

As this is greater than the COC i.e. 14.33% > 12%, the Financial Manager can conclude that the project is financially viable, but can also communicate to interested parties that the rate of return on the project is 14.33% p.a.

3.2   Iterations of Linear Interpolation

The accuracy of the IRR approximation i.e. how close IRR_e is to IRR, will depend on the curvilinear distances between (IRR, 0) and (r_1, NPV_1), and (IRR, 0) and (r_2, NPV_2), where as these distances decrease, the accuracy of the IRR approximation increases. This means that the accuracy of the IRR approximation can be increased by undertaking another iteration of the linear interpolation process i.e. finding another pair of discount rates that result in positive and negative NPVs that are each closer to zero. This can be seen in Figure 3, where r_3 is a discount rate that generates NPV_3 (a positive NPV closer to zero), and r_4 is a discount rate that generates NPV_4 (a negative NPV closer to zero).
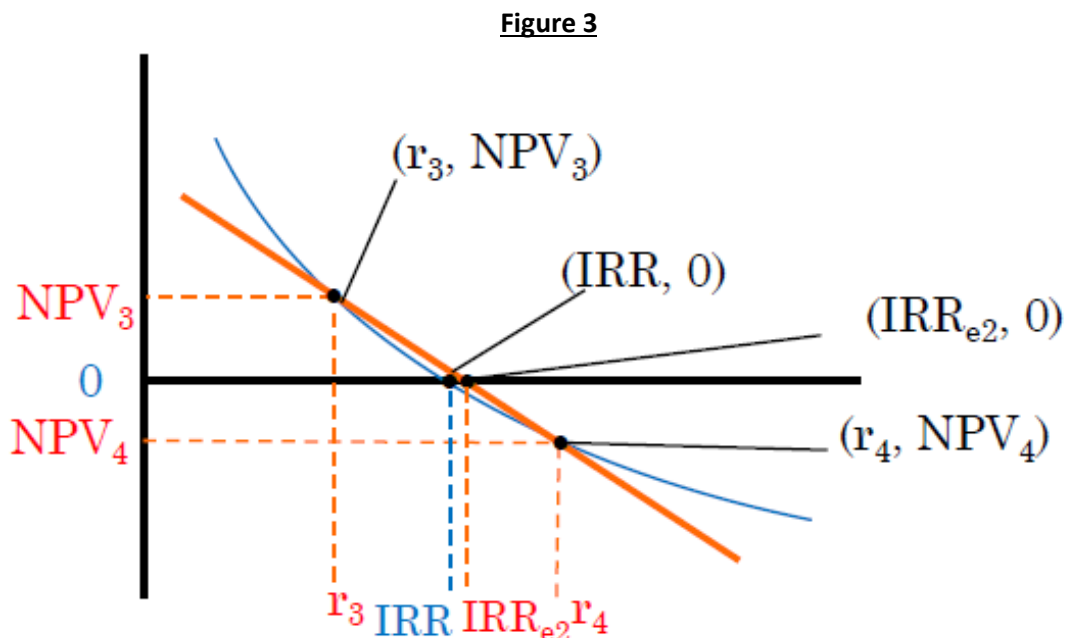
Figure 3

When we draw a straight line through the resulting points of (r_3, NPV_3) and (r_4, NPV_4), we see that the point where the line crosses the X-axis, let's call it $(IRR_{e2}, 0)$, is closer to $(IRR, 0)$ than $(IRR_{e1}, 0)$ was in Figure 2. That is, the second iteration has produced a more accurate approximation of IRR. This is because the section of the curve between (r_3, NPV_3) and (r_4, NPV_4) in Figure 3 is closer to being a straight line than the section of the curve between (r_1, NPV_1) and (r_2, NPV_2) in Figure 2. Were we to undertake a third iteration of the process, we would produce an even more accurate approximation of IRR. Eventually, by undertaking enough iterations, our approximation of IRR will be so close to the true IRR, that the difference can be considered negligible. Returning to Example 2 above, we can see the improved accuracy of the IRR approximation if we undertake a second iteration of linear interpolation using discount rates of 13% and 15% as follows:

$$-€2,000,000 + \frac{€600,000}{1.13^1} + \frac{€800,000}{1.13^2} + \frac{€700,000}{1.13^3} + \frac{€650,000}{1.13^4} = €41,283$$

$$-€2,000,000 + \frac{€600,000}{1.15^1} + \frac{€800,000}{1.15^2} + \frac{€700,000}{1.15^3} + \frac{€650,000}{1.15^4} = -€41,445$$

$$IRR_{e1} = 13\% + \frac{(15\% - 13\%) \times €41,283}{€41,283 - -€41,445} = 13.998\%$$

We can check the accuracy of our approximation by plotting a graph in Excel. Taking discount rates from 5% to 20%, calculating the resulting NPVs and plotting them on an XY-axis, we obtain the following:
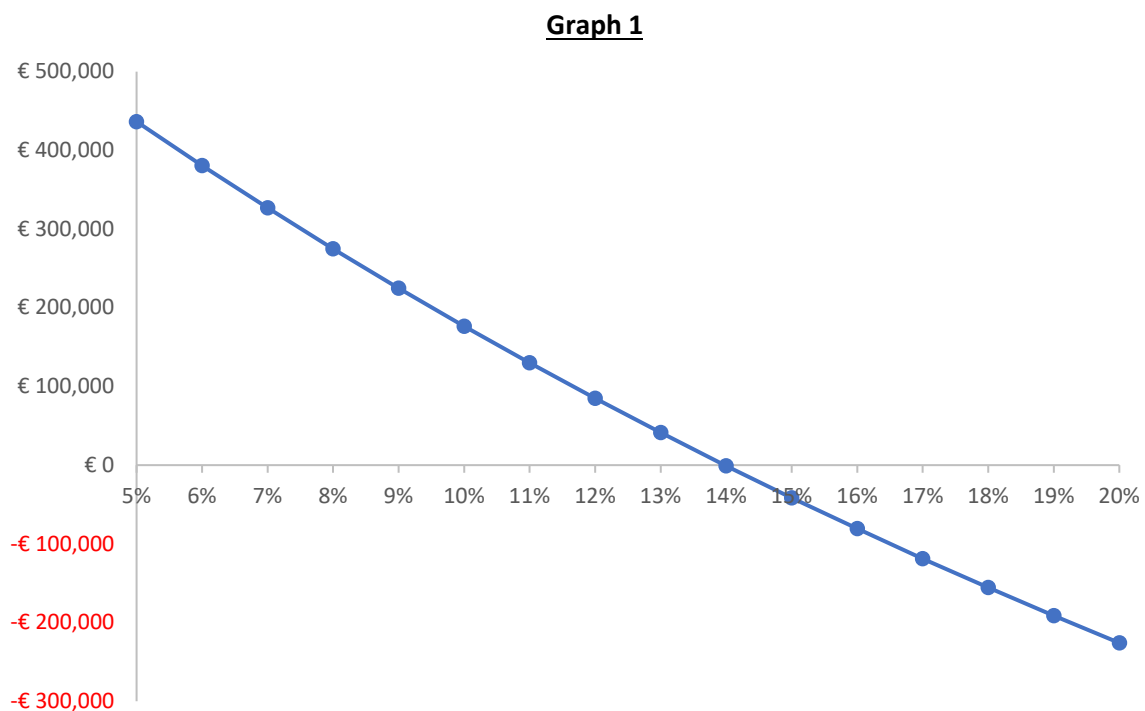
Graph 1

It is clear from the graph that the IRR is indeed very close to 13.998%. We can further confirm this by using the IRR function in Excel, which calculates the IRR of a stream of cash flows accurate to 13 decimal places. Using the IRR function, we find the IRR is approximately equal to 13.9811939177265%!

4.0 Net Present Value v Internal Rate of Return

It is clear from the above explanations and examples that the NPV and IRR methods are closely related, and one might assume that the IRR method is simply a variation of the NPV method that ultimately results in the same investment decision. However, in reality there a number of key differences between the two methods that Financial Managers must be aware of when using either one to make an investment decision.

4.1 Absolute Return and Relative Return

One of the key differences between the NPV and IRR methods is the manner in which project return is expressed. The NPV method expresses the return on a project in absolute terms, where a project's NPV represents the monetary amount added to shareholder value in excess of that required to compensate for the riskiness of the project. On the other hand, the IRR expresses the return on a project in relative terms, where a project's IRR represents the average annual compound rate of return earned on a project's capital outlay. In finance textbooks, this difference is often cited as the main reason that Financial Managers choose to appraise projects using the IRR method, as a relative return expressed as a percentage may be easier to communicate to a project's stakeholders, particularly if they do not have a finance background. However, a relative return can also have disadvantages, particularly when Financial Managers are seeking to compare the financial viability of multiple projects.

4.1.1 IRR and Project Scale

Consider a scenario where a Financial Manager has three investment options to choose from: Project A, Project B and Project C. Each project has a 9% COC, and each would equally aid the firm in achieving its objectives. However, each requires a different capital outlay and generates different annual cash inflows over their three-year duration, as seen in the table below:

Year	0	1	2	3	IRR	NPV @ 9%
Project A	-€5,000,000	€1,800,000	€2,000,000	€2,600,000	12.57%	€342,413
Project B	-€5,950,000	€2,000,000	€2,500,000	€3,100,000	12.32%	€382,831
Project C	-€6,250,000	€2,200,000	€2,600,000	€3,150,000	12.25%	€389,095

If the Financial Manager based the investment decision on the relative IRRs of the projects, they would choose Project A as it has the highest IRR. However, both Project B and Project C have higher NPVs and would therefore add greater shareholder value. This simple example illustrates that because IRR is a relative measure of return, it fails to account for the scale of each project. That is, under certain conditions[7], it may be preferable to earn a lower rate of return on a larger capital outlay, as the absolute return generated is greater.[8]

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[7] Specifically, when capital is not rationed, and projects can be considered as once off investments.
[8] When capital is rationed and / or projects cannot be considered as once off investments, the investment decision becomes more complicated, but the NPV method can be adapted to account for these complications and still result in shareholder wealth maximisation.

4.1.2 IRR and Project Duration

Consider again a scenario where a Financial Manager has three investment options to choose from: Project A, Project B and Project C. Each project has a 9% COC, and each would equally aid the firm in achieving its objectives. Furthermore, each project has the same initial capital outlay, but the durations and annual cash inflows are different across the three projects, as seen in the table below:

Year	0	1	2	3	4	IRR	NPV @ 9%
Project A	-€5,000,000	€2,800,000	€3,400,000			15.09%	€430,519
Project B	-€5,000,000	€2,000,000	€2,200,000	€2,400,000		14.77%	€539,799
Project C	-€5,000,000	€1,200,000	€1,700,000	€2,000,000	€2,300,000	14.63%	€705,518

Again, on the basis of the IRR method, Project A appears to offer the highest return as it has the highest IRR. However, again, Project A would not maximise shareholder value, as both Projects B and C have higher NPVs. This simple example illustrates another shortcoming of relative measures of return i.e. they fail to account for the duration of each project. That is, under certain conditions[9], it may be preferable to earn a lower rate of return over a longer period of time, as the absolute return generated is greater.[10]

4.2 Rate of Reinvestment

One of the strengths of the NPV and IRR methods relative to other investment appraisal methods such as the Payback Period or Accounting Rate of Return is that the NPV and IRR methods take account of the time value of money. That is, they recognise that cash inflows received sooner in time can be reinvested for longer and hence are worth more than cash inflows received later in time. However, there is a key difference between the manner in which each method accounts for the time value of money. To understand this difference, consider the following project that has a COC of 15%:

Year	0	1	2	3	IRR	NPV @ 15%
Cash Flow	-€4,000,000	€1,000,000	€2,000,000	€3,000,000	19.44%	€354,401

The NPV of this project is calculated by discounting back the future cash flows to present values using a discount rate of 15% as follows:

$$-€4,000,000 + \frac{€1,000,000}{1.15^1} + \frac{€2,000,000}{1.15^2} + \frac{€3,000,000}{1.15^3} = €354,401$$

This calculation assumes that as each cash inflow is received, it will be reinvested at a rate of 15% p.a. Given that this project is generating a return greater than 15% p.a., this is not an unrealistic assumption to make i.e. the firm is likely to have other investment opportunities available that can return at least 15% p.a.

The IRR of the project is calculated by identifying the discount rate that results in a zero NPV, which through using linear interpolation, can be shown to be 19.44%:

$$€0 = -€4,000,000 + \frac{€1,000,000}{1.1944^1} + \frac{€2,000,000}{1.1944^2} + \frac{€3,000,000}{1.1944^3}$$

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[9] Specifically, when capital is not rationed, and the duration of a project does not impact a firm's ability to undertake subsequent projects.

[10] When capital is rationed and / or projects with shorter durations facilitate the undertaking of subsequent projects, the investment decision becomes more complicated, but again the NPV method can be adapted accordingly to ensure shareholder wealth maximisation.

This calculation assumes that as each cash inflow is received, it will be reinvested at a rate of 19.44%. Such an assumption may be unrealistic, if the COC of 15% is indicative of the return likely to be made when reinvesting in other investment opportunities.

This example illustrates the difference between how the two methods account for the time value of money. The NPV method assumes cash inflows can be reinvested at the project's COC, whilst the IRR method assumes that cash inflows can be reinvested at the project's IRR. The former assumption can usually be considered to be realistic, unless there is evidence to the contrary, whilst the latter assumption may be unrealistic, particularly in the case of a once-off highly lucrative investment opportunity. Hence, in certain circumstances, the time value of money assumption upon which the IRR is calculated may result in the IRR method providing an exaggerated view of a project's financial viability.

4.3   Unconventional Cash Flows

All the projects outlined above can be classified as having conventional cash flows where there is a single large capital outlay in year 0, and every subsequent cash flow is a cash inflow. However, some projects can have unconventional cash flows where additional capital outlays may be required in future years[11] e.g. the signs of the project's cash flows overtime might show the following pattern: - + + - + + -.[12] In such circumstances, the nature of the relationship between a project's NPV and its discount rate can change significantly. To see how this can happen, consider the following project with unconventional cash flows and a COC of 15%:

Year	0	1	2	3	4
Cash Flow	-€9,600,000	€12,800,000	-€4,500,000	€19,500,000	-€19,000,000

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[11] Examples of projects that can have unconventional cash flows include mining operations or the construction and operation of powerplants, where significant additional investment may be required after several years of operations, or where upon project cessation, significant costs are incurred in shutting down operations.

[12] Note that when there are future cash outflows, taking account of the time value of money involves recognising that each cash outflow must be financed, and hence cash outflows earlier in time cost more that cash outflows later in time.

If we take discount rates from 8% to 26%, calculate the resulting NPVs and plot them on an XY-axis, we obtain the following graph:
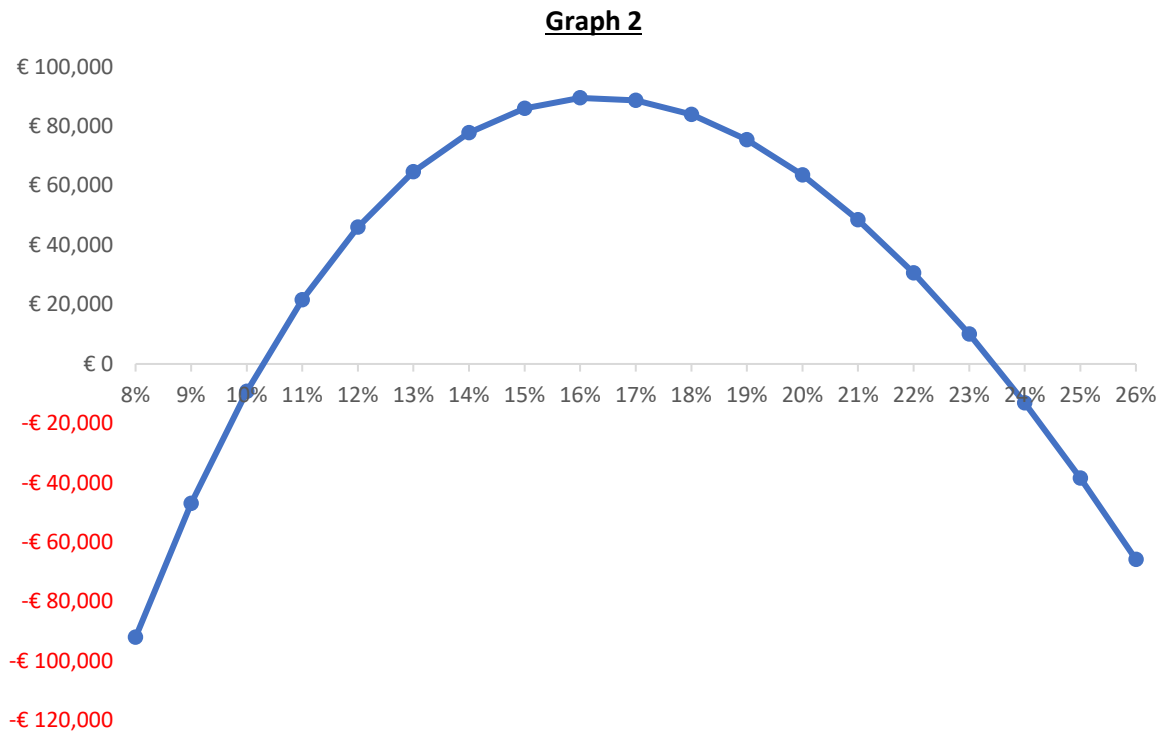
Graph 2

It is clear that the relationship between NPV and discount rate presented in Graph 2 is very different to the relationship between NPV and discount rate presented in Graph 1. In particular, the degree of curvature of the curve in graph 2 is far more pronounced than that of the curve in graph 1, with the end result being that it crosses the x-axis in two places. This has two important implications for using the IRR method to appraise the project.

4.3.1 Multiple IRRs

The IRR of a project is the discount rate that results in a zero NPV. With a conventional cash flow project, there is only one IRR. However, with an unconventional cash flow project, there may be two IRRs[13], as is the case in the preceding example. In such circumstances, the manner in which the IRR method is used to assess financial viability changes. For a conventional cash flow project, if IRR > COC, a project is financially viable. However, for an unconventional cash flow project with two IRRs, as shown above, a project is financially viable if $IRR_1 < COC < IRR_2$, where $IRR_1 < IRR_2$. Based on the graph above, this project would be deemed financially viable as it can be clearly seen that $IRR_1 < 15\% < IRR_2$. Whilst this decision rule is not necessarily difficult to apply, it may be far more difficult to communicate to a stakeholder without the necessary background in finance[14]. Thus, the one advantage that the IRR method is believed to have over the NPV method no longer applies when there are two IRRs. On the other hand, the manner in which the NPV method is used to assess the financial viability remains the same i.e. the project is financially viable if the NPV is positive, regardless of the nature of the cash

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[13] Unconventional cash flow projects do not always have two IRRs. This will depend on the frequency and scale of the future cash outflows, where projects with more frequent and relatively large future cash outflows will tend to have two IRRs. It is possible for unconventional cash flow projects to have three or more IRRs, but the pattern of cash flows needed for this to occur is unlikely to be observed in real world projects.

[14] This decision rule can be interpreted as follows: If the rate at which the project's inflows can be reinvested, and the project's outflows can be financed, is greater than IRR_1 but less than IRR_2, the project is financially viable.

flows.[15] In the case of the above project, given a COC of 15%, the project has a positive NPV of €86,043 and thus is financially viable.

4.3.2   Using Linear Interpolation

The linear interpolation method, as outlined in section 3.1, is appropriate for approximating the IRR of a conventional cash flow project due to the nature of the relationship between NPV and discount rate.[16] With an unconventional cash flow project, the nature of the relationship between NPV and discount rate is often significantly different. This difference means that, if used incorrectly, linear interpolation can lead a decision maker to erroneously conclude that a project is financially viable or unviable. Consider again the unconventional cash flow project above. If we did not graph this project's NPV against its discount rate, we would not know for sure that it has two IRRs. We might therefore conclude that by finding a discount rate that results in a positive NPV and a discount rate that results in a negative NPV, we could estimate the IRR using linear interpolation. For example, discount rates of 12% and 25% would result in NPV's of €46,070 and -€38,400, respectively. Using the linear interpolation formula, we would estimate an IRR of 19.09%. However, the graph clearly shows there is no IRR close to 19.09%, and thus we would have erroneously deemed the project to be financially viable i.e. 19.09% > 15%. Even if we used the IRR function in Excel, we would only find one IRR, as the method Excel uses to find the IRR of a stream of cash flows can only identify one IRR at a time.[17] We might then have used this IRR as the basis of the investment decision, and again erroneously concluded that the project was financially viable or unviable, depending on which IRR we found.

Fortunately, there is a simple way to prevent this mistake being made – plot a graph! A graph allows us to see if there are two IRRs, and we can use linear interpolation to find each IRR. To find the lower of the two IRRs above, we could use discount rates of 9% and 12% to find NPVs of -€46,942 and €46,070, respectively, and use one iteration of linear interpolation to identify 10.51% as IRR_1. To find the higher of the two IRRs above, we could use discount rates of 22% and 25% to find NPVs of €30,624 and -€38,400, respectively, and use one iteration of linear interpolation to identify 23.33% as IRR_2. We could then correctly conclude that as 10.51% < 15% < 23.33%, the project is financially viable. Alternatively, we could use the IRR function in Excel and first enter a guess of 10%, and Excel would return an IRR of 10.2787663640472%. And we could use the IRR function for a second time with a guess of 23%, and Excel would return an IRR of 23.4468850628756%. Again, we can safely say that the COC is between these two rates, and the project is financially viable.

5.0 Conclusion

The NPV and IRR methods are used in practice by Financial Managers to make investment decisions. The methods are based on a similar principal, whereby a project must generate a rate of return greater than the cost of capital for it to be financially viable. However, the manner in which this financial viability criterion is tested varies between the two methods, and it is this variation that results in the IRR method suffering from a number of shortcomings relative to the NPV method. Financial Managers need to be cognisant of these shortcomings when utilising the IRR method, and when in doubt, should rely on the NPV method when making investment decisions.

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[15] With unconventional cash flow projects, the NPV method assumes that future cash inflows can be reinvested at the COC and future cash outflows can be financed at the COC.

[16] That is, as one isolates an increasingly shorter section of the curve that includes the IRR, the closer the section approximates a straight line.

[17] The IRR function in Excel asks for a "guess" i.e., a discount rate at which to begin approximating the IRR. For an unconventional cash flow project with two IRRs, the Excel function will only identify the IRR closest to the discount rate entered as the guess.